Esercizi e ripasso sulle formule goniometriche

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$\cos\left(\frac{7}{12}\pi\right) = ?$

$\frac{7}{12}\pi = \frac{1}{4}\pi + \frac{1}{3}\pi = \frac{\pi}{4} + \frac{\pi}{3}$

$\cos\left(\frac{7}{12}\pi\right) = \cos\left(\underset{\alpha}{\frac{\pi}{4}} + \underset{\beta}{\frac{\pi}{3}}\right) =$

$\cos\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{3}\right) - \operatorname{sen}\left(\frac{\pi}{4}\right)\cdot\operatorname{sen}\left(\frac{\pi}{3}\right) =$

$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} =$

$= \frac{\sqrt{2}-\sqrt{6}}{4}$

$$\tan\left(\frac{7}{12}\pi\right) = \tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right) =$$

$$= \frac{\tan\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{4}\right)}{1-\tan\left(\frac{\pi}{3}\right)\cdot\tan\left(\frac{\pi}{4}\right)} =$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} = \frac{1+\sqrt{3}}{1-\sqrt{3}} =$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} =$$

$$= \frac{(1+\sqrt{3})^2}{1-(\sqrt{3})^2} = \frac{1+2\sqrt{3}+3}{-2} =$$

$$= \frac{4+2\sqrt{3}}{-2} = \frac{2(2+\sqrt{3})}{-2} =$$

$$= -2 - \sqrt{3} .$$

$$\operatorname{sen}\left(\frac{13}{12}\pi\right) = ?$$

$$\frac{13}{12}\pi = \frac{1}{3}\pi + \frac{3}{4}\pi$$

$$\operatorname{sen}\left(\frac{13}{12}\pi\right) = \operatorname{sen}\left(\frac{\pi}{3} + \frac{3}{4}\pi\right) =$$

$$\operatorname{sen}\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{3}{4}\pi\right) + \operatorname{Sen}\left(\frac{3}{4}\pi\right)\cdot\cos\left(\frac{\pi}{3}\right) =$$

$$\frac{\sqrt{3}}{2}\cdot\underbrace{\cos\left(\pi - \frac{\pi}{4}\right)}_{-\cos\left(\frac{\pi}{4}\right)} + \underbrace{\operatorname{Sen}\left(\pi - \frac{\pi}{4}\right)}_{\operatorname{Sen}\left(\frac{\pi}{4}\right)}\cdot\frac{1}{2} =$$

$$\frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} =$$

$$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}.$$

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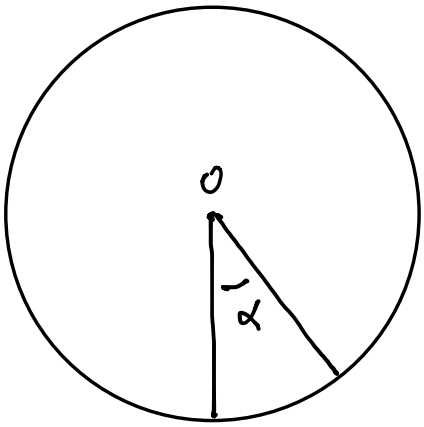

24 fette:

$$\alpha = \frac{2\pi}{24} = \frac{\pi}{12}$$

$$\alpha = \frac{\pi}{12} = 15° = \frac{\pi}{3} - \frac{\pi}{4}$$

$$\operatorname{sen}\left(\frac{\pi}{12}\right) = \operatorname{sen}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) =$$

$$\operatorname{sen}\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{4}\right) - \operatorname{sen}\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{3}\right) =$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

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$$\cos\left(\alpha - \frac{11}{6}\pi\right) - \operatorname{sen}\left(\frac{5}{3}\pi + \alpha\right) =$$

$$\cos(\alpha) \cdot \cos\left(\frac{11}{6}\pi\right) + \operatorname{sen}(\alpha) \cdot \operatorname{sen}\left(\frac{11}{6}\pi\right) -$$

$$- \left[\operatorname{sen}\left(\frac{5}{3}\pi\right) \cdot \cos\alpha + \operatorname{sen}(\alpha) \cdot \cos\left(\frac{5}{3}\pi\right)\right] =$$

$\cos\left(\frac{\pi}{6}\right)$ $\qquad$ $-\operatorname{sen}\frac{\pi}{6}$

$$\cos(\alpha) \cdot \cos\left(2\pi - \frac{\pi}{6}\right) + \operatorname{sen}(\alpha) \operatorname{sen}\left(2\pi - \frac{\pi}{6}\right) -$$

$$- \left[\operatorname{sen}\left(2\pi - \frac{\pi}{3}\right) \cdot \cos(\alpha) + \operatorname{sen}\alpha \cdot \cos\left(2\pi - \frac{\pi}{3}\right)\right] =$$

$-\operatorname{sen}\left(\frac{\pi}{3}\right)$ $\qquad$ $\cos\left(\frac{\pi}{3}\right)$

$$\cos(\alpha) \cdot \frac{\sqrt{3}}{2} + \operatorname{sen}(\alpha) \cdot \left(-\frac{1}{2}\right) -$$

$$-\left(-\frac{\sqrt{3}}{2}\right)\cdot\cos\alpha-\operatorname{sen}(\alpha)\cdot\frac{1}{2}=$$

$$\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}\operatorname{sen}\alpha+\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}\operatorname{sen}\alpha$$

$$\sqrt{3}\cos(\alpha)-\operatorname{sen}(\alpha)\,.$$

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DATI: $\operatorname{sen}\alpha=\frac{1}{3}$

$\cos\beta=\frac{2}{3}$

α,β ACUTI ↓ I QUADR.

CALCOLA: $\cos(\alpha+\beta)$

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\operatorname{sen}\alpha\cdot\operatorname{sen}\beta$$

$\cos\beta=\frac{2}{3}$, $\operatorname{sen}\alpha=\frac{1}{3}$

$$\operatorname{sen}\alpha = \frac{1}{3} \rightarrow \cos\alpha = \pm\sqrt{1-\operatorname{sen}^2\alpha}$$

I QUADR.

$$\cos\alpha = \sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\cos\beta = \frac{2}{3} \rightarrow \operatorname{sen}\beta = \pm\sqrt{1-\cos^2\beta}$$

I QUADR.

$$\operatorname{sen}\beta = \pm\sqrt{1-\left(\frac{2}{3}\right)^2} = \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}}$$

$$\pm\frac{\sqrt{5}}{3}.$$

$$\cos(\alpha+\beta) = \underbrace{\cos\alpha}_{\frac{2\sqrt{2}}{3}} \cdot \underbrace{\cos\beta}_{\frac{2}{3}} - \underbrace{\operatorname{sen}\alpha}_{\frac{1}{3}} \cdot \underbrace{\operatorname{sen}\beta}_{\frac{\sqrt{5}}{3}}$$

$$= \frac{4\sqrt{2}}{9} - \frac{\sqrt{5}}{9} = \frac{4\sqrt{2}-\sqrt{5}}{9}.$$

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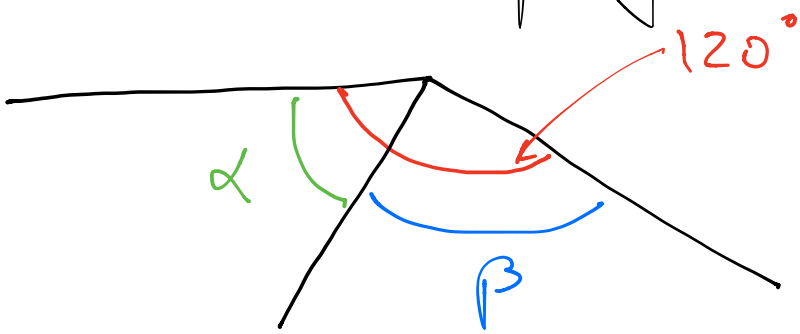

$\cos(\alpha) = \frac{4}{5}$ $\quad \operatorname{sen}(\beta) = ?$

$\beta = 120 - \alpha$

$\operatorname{sen}(\beta) = \operatorname{sen}(120 - \alpha) =$

$\operatorname{sen}(120) \cdot \underbrace{\cos(\alpha)}_{4/5} - \underbrace{\operatorname{sen}(\alpha)}_{?} \cdot \cos(120)$

$0 < \alpha < 90° \Rightarrow \operatorname{sen}(\alpha) = +\sqrt{1 - \cos^2\alpha}$

$= +\sqrt{1 - \left(\frac{4}{5}\right)^2}$

$$\operatorname{sen}(\alpha) = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\operatorname{sen}(120^\circ) = \operatorname{sen}(180 - 60) = \operatorname{sen}(60)$$
$$\pi \qquad \frac{\pi}{3} \qquad \frac{1}{\sqrt{3}/2}$$

$$\cos(120) = \cos(180 - 60) = -\cos 60 = -\frac{1}{2}$$

$$\underbrace{\operatorname{sen}(120)}_{\sqrt{3}/2} \cdot \underbrace{\cos(\alpha)}_{4/5} - \underbrace{\operatorname{sen}(\alpha)}_{3} \cdot \underbrace{\cos(120)}_{-1/2} =$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{4}{5} - \frac{3}{5}\left(-\frac{1}{2}\right) =$$

$$= \frac{4\sqrt{3}}{10} + \frac{3}{10} = \frac{4\sqrt{3}+3}{10}$$

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$$\frac{1-\cos(2\alpha)}{1+\cos(2\alpha)} \cdot \frac{1}{\tan(\alpha)} =$$

$$\cos(2\alpha) = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 2\cos^2\alpha - 1 \\ 1 - 2\sin^2\alpha \end{cases}$$

$$= \frac{1-(1-2\sin^2\alpha)}{1+(2\cos^2\alpha-1)} \cdot \frac{1}{\frac{\sin\alpha}{\cos\alpha}} =$$

$$= \frac{1-1+2\sin^2\alpha}{1+2\cos^2\alpha-1} \cdot \frac{\cos\alpha}{\sin\alpha} =$$

$$= \frac{2\operatorname{sen}^2\alpha}{2\cos^2\alpha} \cdot \frac{\cos\alpha}{\operatorname{sen}\alpha} =$$

$$= \frac{\operatorname{sen}\alpha}{\cos\alpha} = \tan(\alpha) \quad \checkmark$$