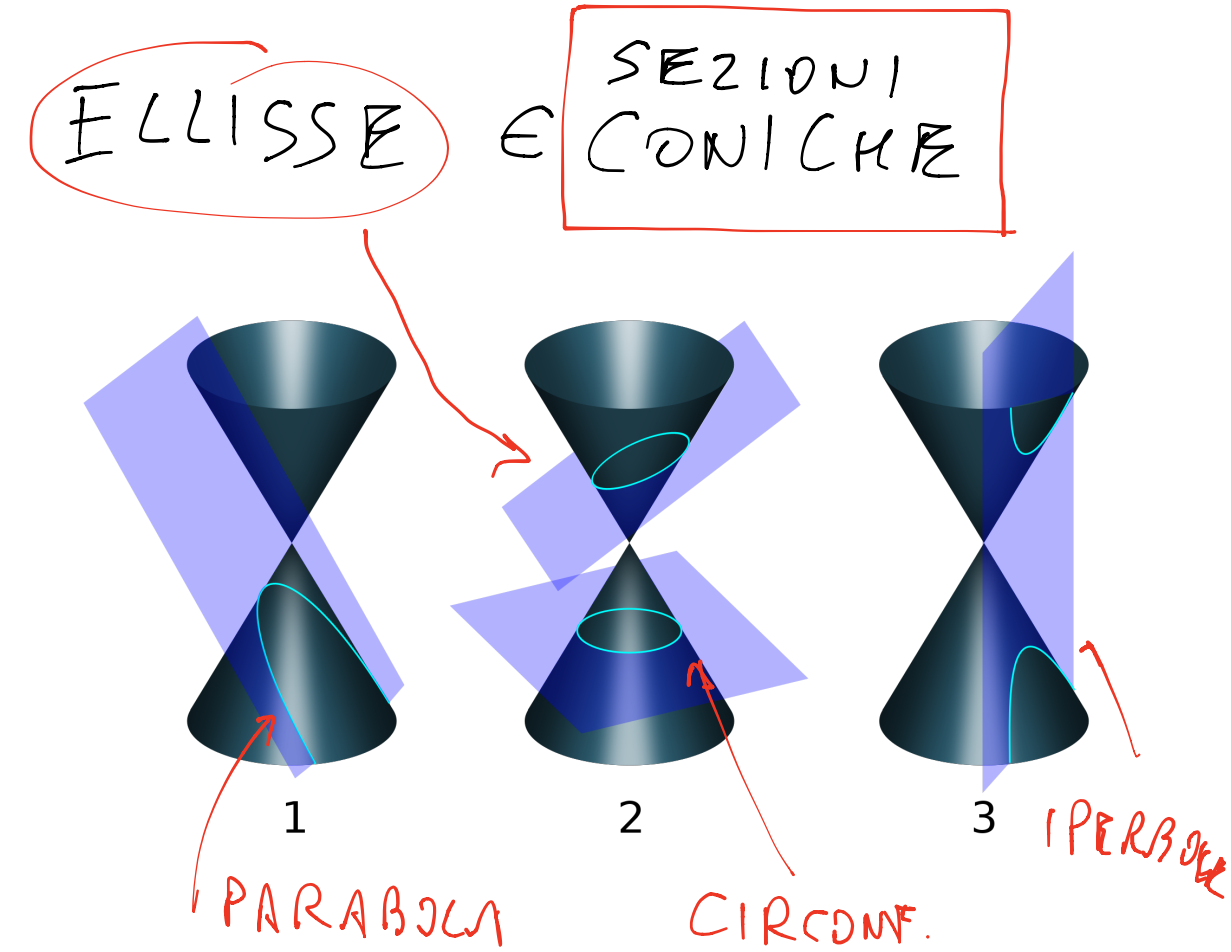
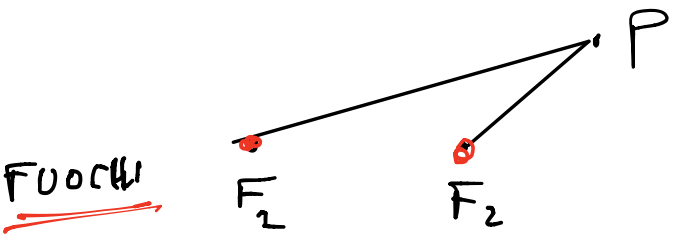
L'ellisse nel piano cartesiano, definizione, proprietà ed esercizi.

AS 2020-21. Prof. A. Pisani



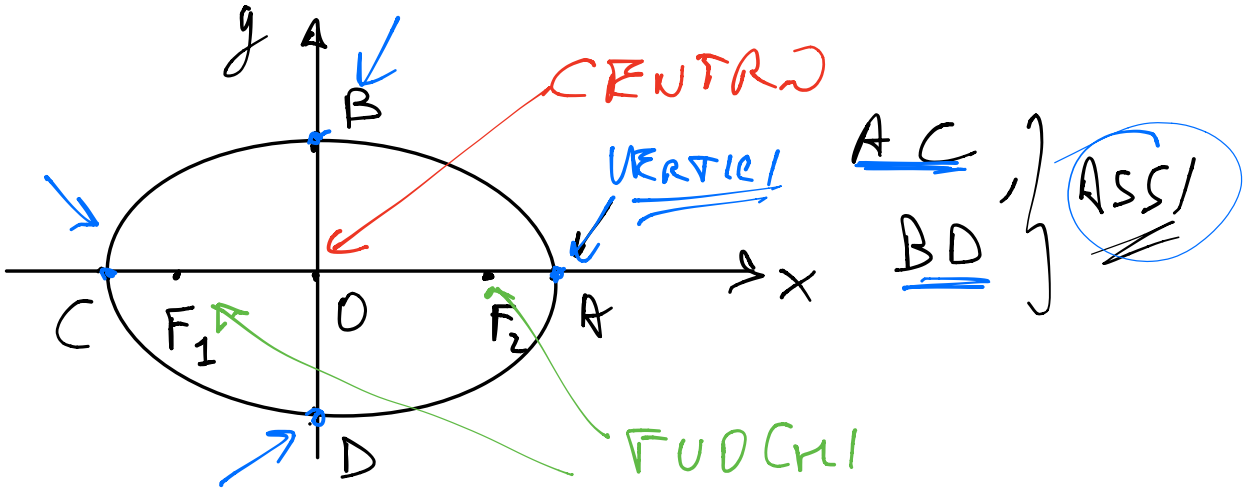
NEL PIANO :



$$\overline{PF_1} + \overline{PF_2} = \text{COST}$$

$\Downarrow$

ELLISSE



$F_1, F_2 \in$ ASSE x $\Rightarrow \overline{AC} > \overline{BD}$

AC ASSE MAGGIORE

BD " MINORE

O CENTRO DELL'ELLISSE

$\overline{OA} = \frac{1}{2}\overline{AC}$ = SEMIASSE MAGG.

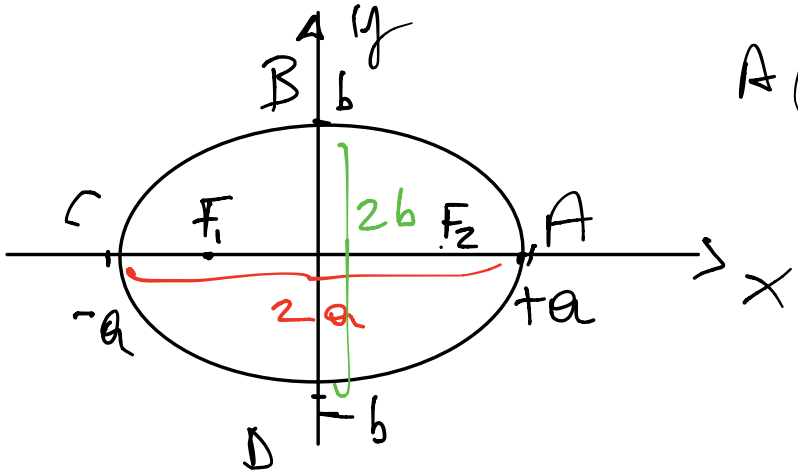
$\overline{OB} = \frac{1}{2}\overline{BD}$ = SEMIASSE MINORE

EQUAZIONE CARTESIANA:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a \neq 0$

$b \neq 0$



$A(a; 0)$

$B(0; b)$

$C(-a; 0)$

$D(0; -b)$

a = SEMIASSE ORIZZ. = $\overline{OA}$

b = " VERTICALE = $\overline{OB}$

$a > b \iff F_1, F_2 \in$ ASSE x

$F_1(-c; 0)$ $F_2(+c; 0)$

$$c = \sqrt{a^2 - b^2}$$

$\overline{F_1F_2} = 2c$

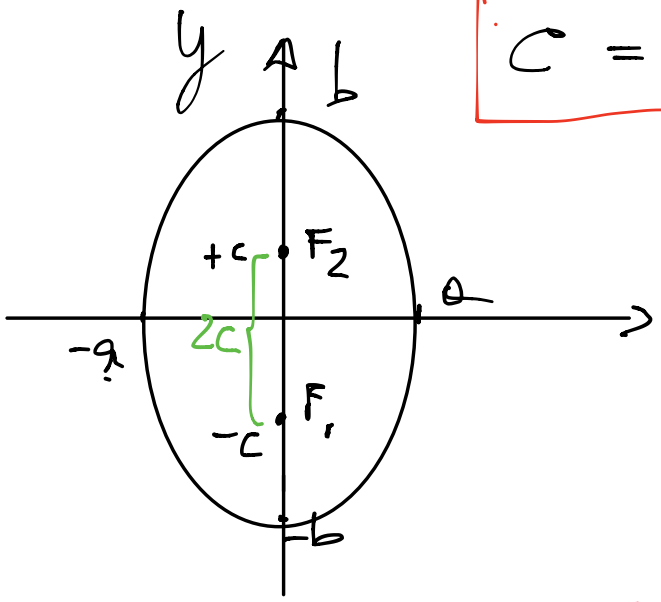
IN GEN..

$$c = \sqrt{(\text{SEMIASSE MAGG})^2 - (\text{SEMIASSE MIN})^2}$$

SE $b > a \Rightarrow F_1, F_2 \in$ ASSE y

SEMIASSE VERT

$$c = \sqrt{b^2 - a^2}$$



$F_1(0; -c)$

$F_2(0; +c)$

ECCENTRICITA' e

$$e = \frac{\text{DIST. FOCALE}}{\text{ASSE MAGGIORE}} < 1$$

$$0 \leq e < 1$$

DIST. FOCALE $= \overline{F_1F_2} = 2c$

$$a > b \Rightarrow e = \frac{2c}{2a} = \frac{c}{a}$$

$$b > a \Rightarrow e = \frac{2c}{2b} = \frac{c}{b}$$

SE $a > b$ $\quad e = \dfrac{c}{a}$

$$= \frac{\sqrt{a^2 - b^2}}{a}$$

$$= \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$= \sqrt{1 - \left(\frac{b}{a}\right)^2} < 1$$

$e = 1 \iff b = 0 \Rightarrow$ DEGENERA SEGMENTO

$e = 0 \Rightarrow \frac{c}{a} = 0 \quad c = 0$

$\Downarrow$

$a = b$

$e = 0 \Rightarrow$ ELLISSE = CIRCONF.

$a = b = r.$

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$$9x^2 + 4y^2 = 36$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

DIVIDO PER 36

$$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a, b > 0$

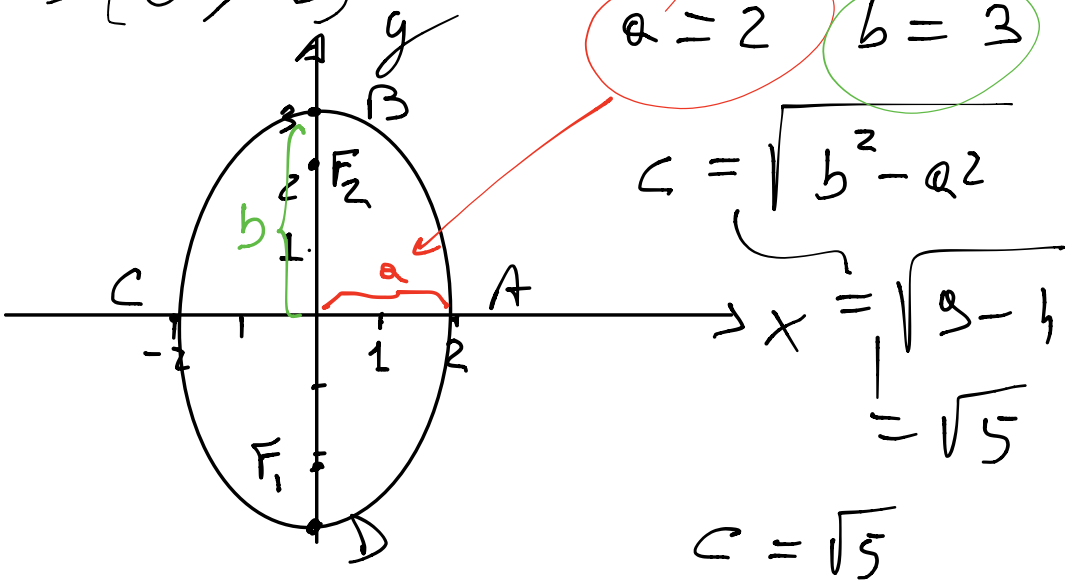
$a^2 = 4 \Rightarrow a = +2$

$b^2 = 9 \Rightarrow b = +3$

$b > a \Rightarrow$ FUOCHI $\in$ ASSE y

$A(2;0)$ $B(0;3)$ $C(-2;0)$

$D(0;-3)$



$a = 2$ $b = 3$

$c = \sqrt{b^2 - a^2}$

$x = \sqrt{9-4}$

$= \sqrt{5}$

$c = \sqrt{5}$

$$F_1(0;-\sqrt{5}) \qquad F_2(0;+\sqrt{5}).$$

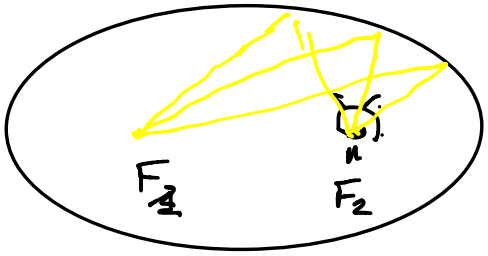
$$e = \frac{\text{DIST. FOC.}}{\text{ASSE MAGG}} = \frac{2c}{2b} = \frac{\sqrt{5}}{3} < 1.$$

→

AREA DELL'ELLISSE:

$$S = \pi \cdot b \cdot a$$

←



ECCENTRICITÀ =

$$\frac{\text{SEMIDIST. FOCALE}}{\text{SEMIASSE MAGG.}}$$

$$e = \begin{cases} \dfrac{c}{a} & a > b \\ \dfrac{c}{b} & b > a \end{cases}$$

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$$\frac{x^2}{16} + y^2 = 1 \iff \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{1}$$

$$a^2 = 16 \Rightarrow a = 4$$

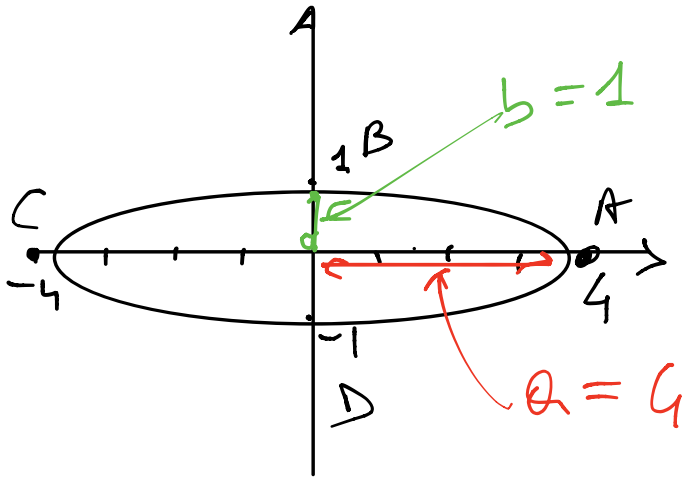
$$b^2 = 1 \Rightarrow b = 1$$

$\boxed{a > b} \Rightarrow$ FUOCHI $\in$ ASSE x

$F_1(-c;0)$ $F_2(c;0)$

$$c = \sqrt{a^2 - b^2} = \sqrt{16 - 1} = \sqrt{15}.$$

$$e = \frac{c}{a} = \frac{\sqrt{15}}{4} < 1.$$



$e \simeq 0.968\ldots$

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FUOCHI E ASSE x $(a > b)$

$b = 2$ $\quad e = \frac{\sqrt{3}}{2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$e = \frac{\sqrt{3}}{2} = \frac{c}{a}$$

$$e = \frac{c}{a} \qquad c = \sqrt{a^2 - b^2} \quad (a > b)$$

$$e = \frac{\sqrt{a^2 - b^2}}{a} \qquad a = ?$$

$$(e)^2 = \left(\frac{\sqrt{a^2 - b^2}}{a}\right)^2$$

$$e^2 = \frac{a^2 - b^2}{a^2}$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - e^2$$

$$\frac{b}{a} = \pm\sqrt{1 - e^2} \qquad \cdot a$$

$$a \cdot \frac{b}{a} = a \cdot \sqrt{1 - e^2}$$

$$\frac{b}{\sqrt{1 - e^2}} = a$$

$$a = \frac{b}{\sqrt{1-e^2}} = \frac{2}{\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$= \frac{2}{\sqrt{1-\frac{3}{4}}} = \frac{2}{\sqrt{\frac{1}{4}}} =$$

$$= \frac{2}{\frac{1}{2}} = 4$$

$a = 4 \qquad b = 2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\rightarrow \boxed{\frac{x^2}{16} + \frac{y^2}{4} = 1} \qquad \cdot 16$$

$$x^2 + 4y^2 = 16$$

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FUOCHI ∈ ASSE y ($b > a$)

$2a = 4$ $\quad$ $2c = 2$

$2a$ → ASSE IN $\overline{AC}$

$2c$ → DIST. FOCALE $\overline{F_1F_2}$

$2a = 4 \rightarrow a = 2$

$2c = 2 \rightarrow c = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad b > a$$

$F_1(0; -c)$ $\quad$ $F_2(0; +c)$

$$c = \sqrt{b^2 - a^2}$$

$$c^2 = b^2 - a^2$$

$$b^2 = c^2 + a^2$$

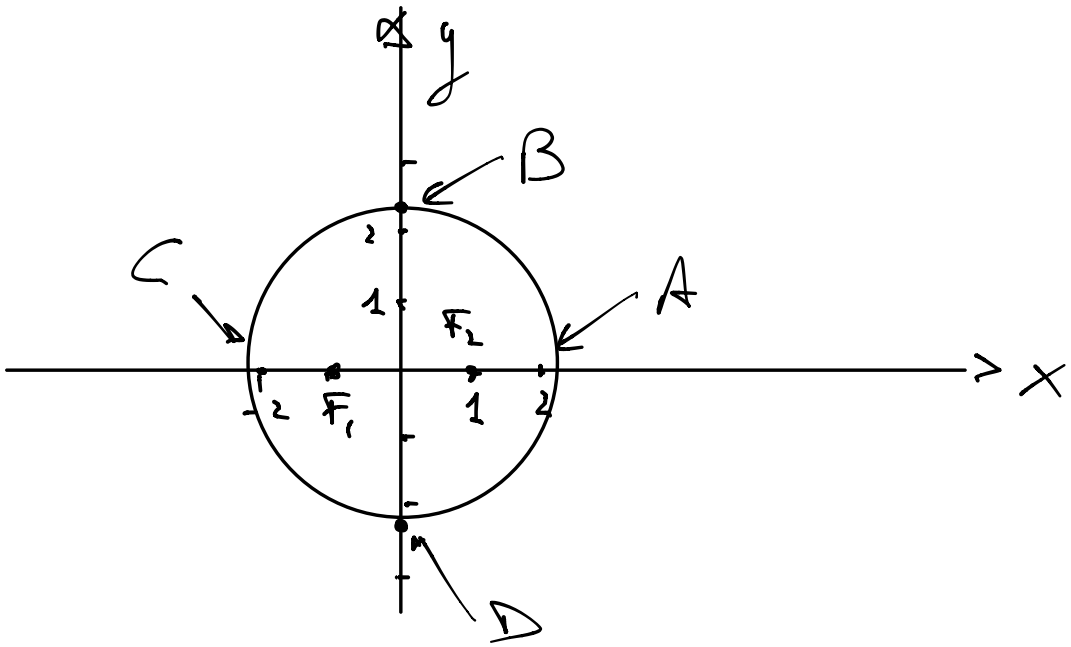
$$b = \sqrt{c^2 + a^2}$$

$$= \sqrt{1 + 4} = \sqrt{5}$$

$$b^2 = 5 \qquad a^2 = 4$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow \boxed{\frac{x^2}{4} + \frac{y^2}{5} = 1}$$

$$\sqrt{5} \simeq 2.23$$



$$e = \frac{c}{b} = \frac{1}{\sqrt{5}} \simeq 0.45$$

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$$y = \pm\sqrt{25 - 4x^2}$$ ← FORMA ESPLICITA

ATT.:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ELLISSE CON CENTRO IN O.

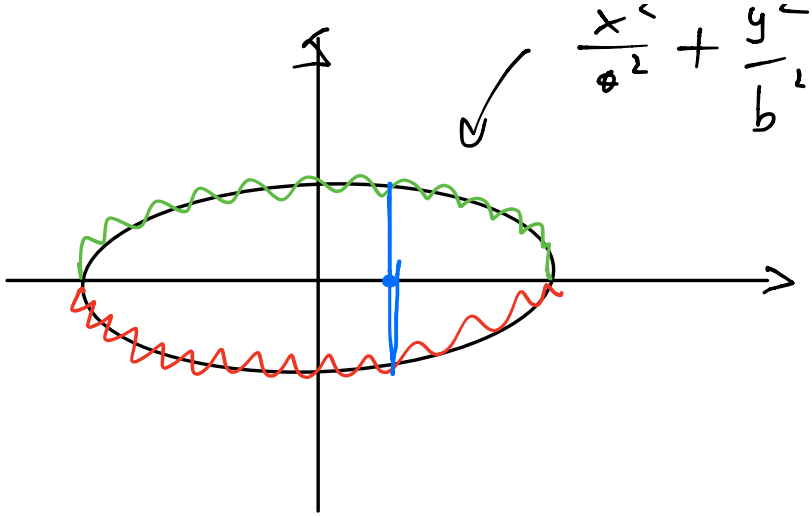
FORMA IMPLICITA $\longrightarrow$ F. ESPL.

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$\frac{y}{b} = \pm \sqrt{1 - \frac{x^2}{a^2}}$$

$$y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$$

$$y = \pm \sqrt{b^2 - \frac{b^2}{a^2} x^2}$$



$$y = \pm\sqrt{25 - 4x^2}$$

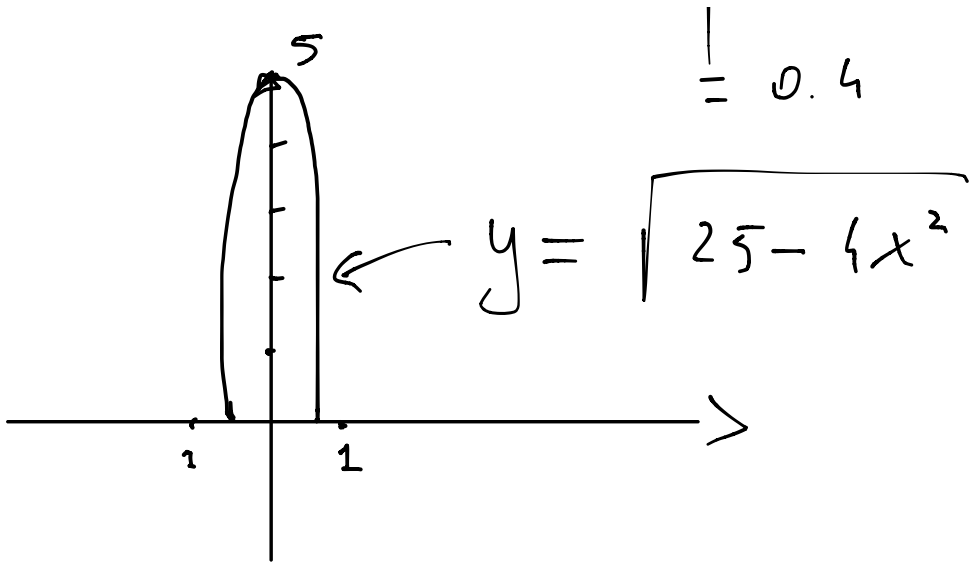
PARTE SUP. ($y > 0$)

$$y = \pm\sqrt{b^2 - \frac{b^2}{a^2}x^2}$$

$$b^2 = 25 \Rightarrow b = 5$$

$$\frac{b^2}{a^2} = 4 \Rightarrow a^2 = \frac{4}{b^2} = \frac{4}{25}$$

$$a = \frac{2}{5} < b$$



$\frac{1}{5} = 0.4$