Esercizi sulla circonferenza

AS 2020-21 prof. A. Pisani

Es 113 pag 317

$C: x^2+y^2+4x-2y=0$

$r: x+3y+4=0$

SISTEMA

$$\begin{cases} x^2+y^2+4x-2y=0 \\ x+3y+4=0 \end{cases}$$

$$\rightarrow \begin{cases} (-3y-4)^2+y^2+4(-3y-4)-2y=0 \\ x=-3y-4 \end{cases}$$

$$9y^2 + 24y + \cancel{16} + y^2 - 12y - \cancel{16} - 2y = 0$$

$$10y^2 + 10y = 0 \quad :10$$

$$y^2 + y = 0$$

$$y(y+1) = 0$$

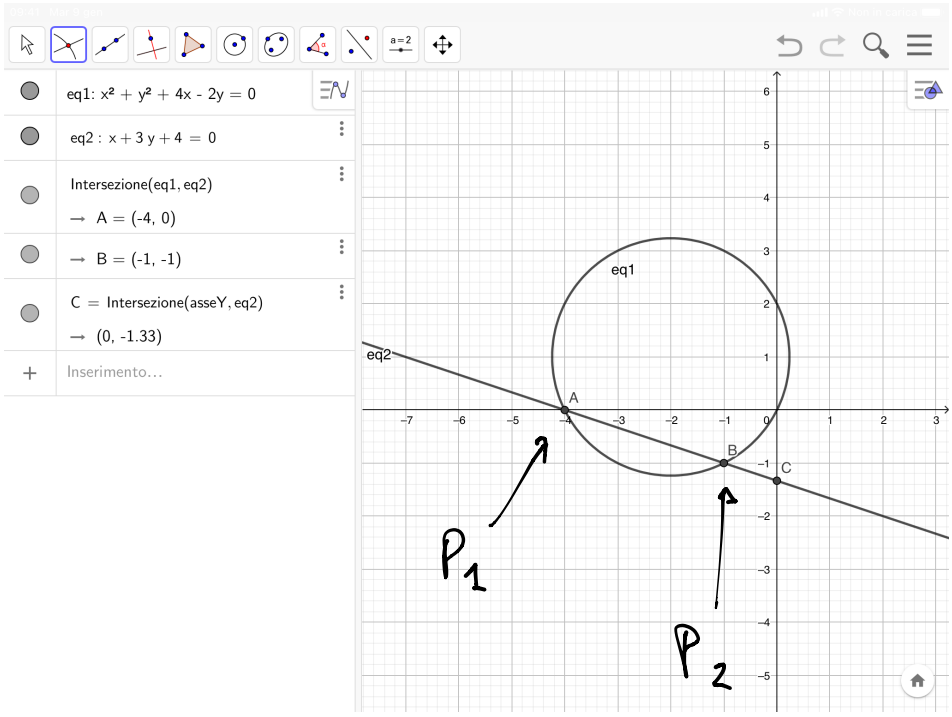
$$y_1 = 0 \rightarrow x_1 = -4$$

$$y_2 = -1 \rightarrow x_2 = -3(-1) - 4$$

$$= +3 - 4 = -1$$

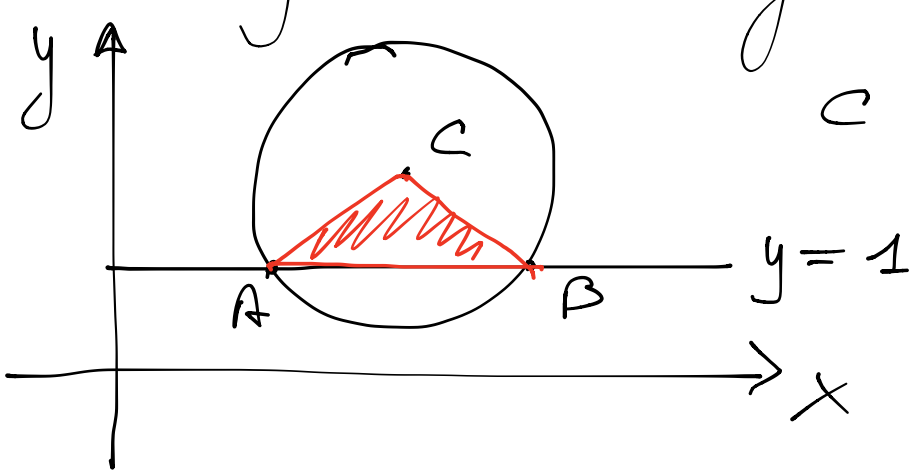
RETTA SECANTE

$P_1(-4;0)$ $P_2(-1;-1)$



Es. 122 pag 318

$$x^2+y^2-8x-6y+17=0$$



C ?

A ?

B ?

$$x^2+y^2-8x-6y+17=0$$

$$x^2+y^2-2\alpha x-2\beta y+\gamma=0$$

$$-2\alpha=-8 \rightarrow \alpha=4$$

$$-2\beta=-6 \rightarrow \beta=3$$

$$C(4;3)$$

$$\begin{cases} x^2+y^2-8x-6y+17=0 \\ y=1 \end{cases}$$

$$x^2+1-8x-6+17=0$$

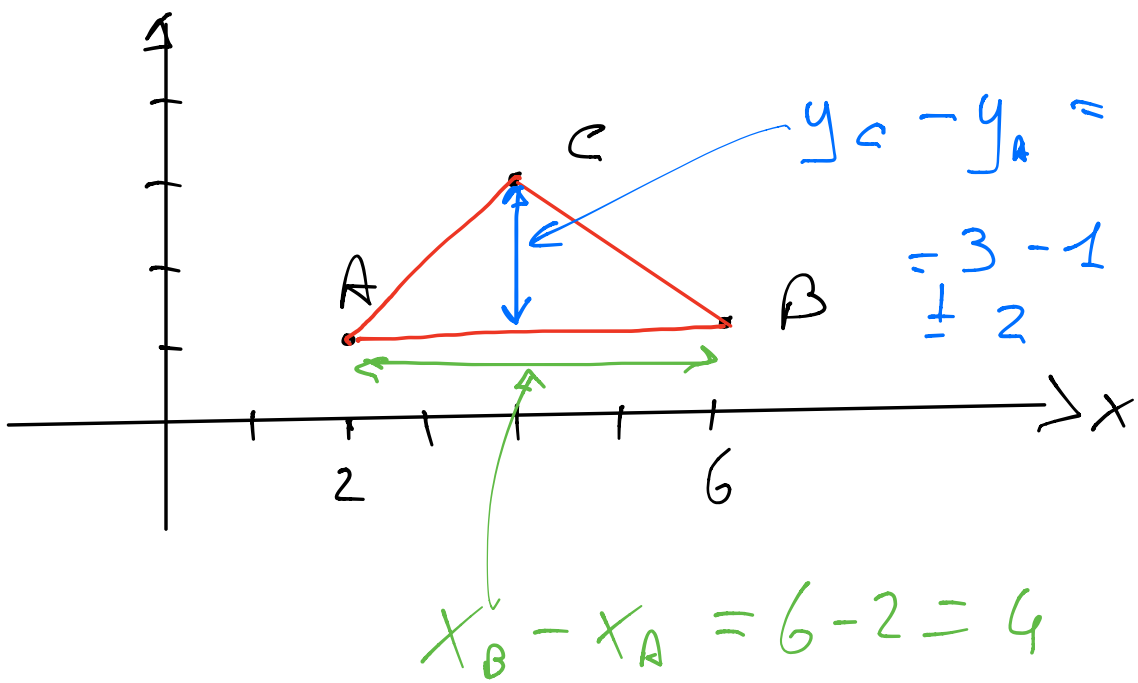
$$\rightarrow x^2-8x+12=0$$

$a = 1 \qquad b = -8 \qquad c = +12$

$$\Delta = 64 - 48 = 16$$

$$x_{1,2} = \frac{8 \pm 4}{2} \begin{cases} 6 \\ 2 \end{cases}$$

$A(2;1) \qquad B(6;1)$



$$\text{Area} = b \cdot h/2 = 4 \cdot 2/2 = 4$$

Es. 124 p. 318

$r: \; y = 3x - 5$

è tangente in P a:

$C: \; x^2 + y^2 - 12x - 6y + 35 = 0$

calcolo $\overline{PO} = ?$

$$\begin{cases} y = 3x - 5 \\ x^2 + y^2 - 12x - 6y + 35 = 0 \end{cases}$$

$$x^2 + (3x-5)^2 - 12x - 6(3x-5) + 35 = 0$$

$$x^2 + 9x^2 - 30x + 25 - 12x - 18x +$$
$$+ 30 + 35 = 0$$

$$10x^2 - 60x + 90 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0 \rightarrow \underline{\underline{x = 3}}$$

$$a = 1 \quad b = -6 \quad c = 9$$

$$\Delta = 36 - 4 \cdot 1 \cdot 9 = 36 - 36 = \underline{\underline{0}}$$

RETTA TANGENTE

$$P(3; 4)$$

$$y = 3x - 5 = 3 \cdot 3 - 5 = 4$$

$$P(3; 4) \qquad O(0; 0)$$

$$\overline{OP} = \sqrt{(x_P - x_O)^2 + (y_P - y_O)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

ES 120 pag 318

$$A, B = r \cap C$$

$$r: \quad y = 2x - 4$$

$$C: \quad x^2 + y^2 + 4x - 4y - 17 = 0$$

$$\overline{AB} = ?$$

$$\begin{cases} y = 2x - 4 \\ x^2 + y^2 + 4x - 4y - 17 = 0 \end{cases}$$

$$x^2 + (2x-4)^2 + 4x - 4(2x-4) - 17 = 0$$

$$x^2 + 4x^2 - 16x + 16 + 4x - 8x + 16 + 17 = 0$$

$$5x^2 - 20x + 15 = 0 \quad :5$$

$$x^2 - 4x + 3 = 0$$

$$a = 1 \qquad b = -4 \qquad c = +3$$

$$\Delta = b^2 - 4ac = 16 - 4 \cdot 1 \cdot 3$$

$$= 16 - 12 = +4$$

$$x_{1,2} = \frac{4 \pm 2}{2} \begin{cases} 3 & \leftarrow x_B \\ 1 & \leftarrow x_A \end{cases}$$

$$x_A = 1 \longrightarrow y_A = 2x_A - 4 = 2 \cdot 1 - 4 = -2$$

$x_B = 3 \rightarrow y_B = 2 \cdot 3 - 4 = +2$

$A(1; -2) \quad B(3; 2)$

$$\overline{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

$$= \sqrt{(3-1)^2 + (2-(-2))^2}$$

$$= \sqrt{2^2 + 4^2}$$

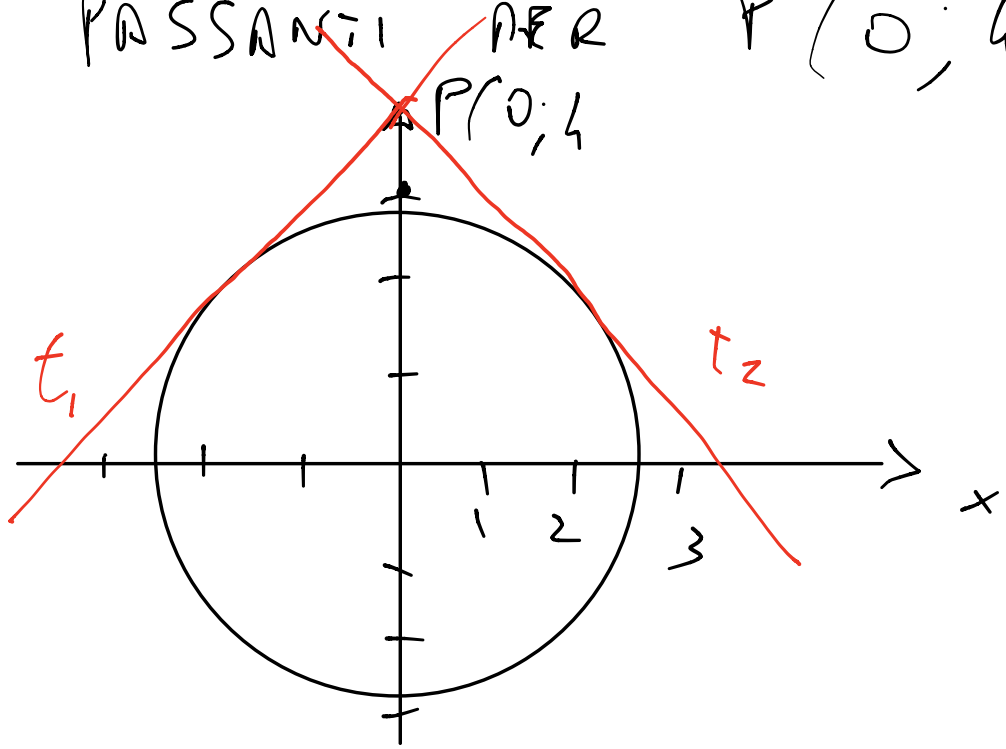
$$= \sqrt{4+16} = \sqrt{20} = \underline{2\sqrt{5}}.$$

ES 135 pag. 320

RETTE TANGENTI

$$C: x^2 + y^2 = 8 \rightarrow \begin{array}{l} C(0;0) \\ r = \sqrt{8} = 2\sqrt{2} \end{array}$$

PASSANTI PER $P(0;4)$



TUTTE LE RETTE PERP

$$P(x_P; y_P): \quad y - y_P = m(x - x_P)$$

$$P(0;4) \rightarrow \quad y - 4 = m(x - 0)$$

$$y = mx + 4$$

?

$$\begin{cases} y = mx + 4 \\ x^2 + y^2 = 8 \end{cases} \rightarrow \Delta = 0$$

$\downarrow$

TANG.

$$x^2 + (mx+4)^2 = 8$$

$$x^2 + m^2x^2 + 8mx + 16 = 8$$

$$(1+m^2)x^2 + 8mx + 8 = 0$$

$$\Delta = (8m)^2 - 4 \cdot 8 \cdot (1+m^2) = 0$$

$$64m^2 - 32 - 32m^2 = 0$$

$$32m^2 - 32 = 0 \quad : 32$$

$$m^2 - 1 = 0$$

$$m^2 = 1$$

$$m = \pm 1 \begin{cases} \nearrow t_1 \\ \searrow t_2 \end{cases}$$

t_1 : $\boxed{y = -x + 4}$ $(m = -1)$

t_2 : $\boxed{y = +x + 4}$ $(m = +1)$